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Quantum phase expectation values of a mesoscopic Josephson junction from quantum current measurements

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Abstract. A simple way to acquire information on the mean values of the phase operators $\sin\varphi$ and $\cos\varphi$ of an ultrasmall Josephson junction prepared in an arbitrary pure or not state is reported. Our proposal exploits the recently predicted occurrence of current spikes in the *I*-*V* characteristic of a mesojunction irradiated by a quantum single-mode low-intensity coherent electromagnetic field. A necessary condition for the validity of our treatment is presented and discussed.

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1 Introduction

Recent experimental and theoretical studies of small capacitance tunnel junctions at low temperatures have revealed a new class of effects as Coulomb blockade, single electron tunneling and Bloch oscillations [1–3]. Several macroscopic quantum effects in Josephson junctions (JJ) have been discussed and some of them have been observed in experiments [4–8]. Advances in nanotechnology and microelectronics should enable the laws of quantum dynamics to be tested at the macroscopic level [9].

The study of these non classical effects is crucial both in order to establish the fundamental limits of conventional devices and because such effects open new perspectives for future applications, including quantum engineering based on superconductive elements. In order to analyze such non classical effects, one should consider all the quantities describing the junction as operators rather than classical variables. The operators corresponding to the variables characterizing a Josephson junction, the phase difference φ and the electric charge q of the geometric capacitance C formed by the junction electrodes, should satisfy the following commutation rule $[\varphi, q] = 2e\mathrm{i}$ as well as the Heisenberg uncertainty relation $\Delta\varphi\ \Delta q \geq e$, usually neglected in the classical theory of the Josephson effects [2,4,10]. The ratio $\frac{E_\mathrm{C}}{2E_\mathrm{J}}$, $E_\mathrm{C} = \frac{(2e)^2}{C}$ and $E_\mathrm{J} = \frac{\hbar I_\mathrm{cr}}{2e}$ being the charging and the Josephson energy respectively, is a reasonable indicator of the crossover point from classical regime to the quantum one. In the classical regime, in fact, the ground state of a JJ is a narrowly peaked wavefunction $\psi(\varphi)$ with a width of the order of $(\frac{E_\mathrm{C}}{2E_\mathrm{J}})^{\frac{1}{4}} \ll 1$, corresponding to very small quantum fluctuations in the phase. By contrast, in the quantum regime, where $E_\mathrm{C} \gg E_\mathrm{J}$, all values of φ are equally probable and we analyze the quantum states of a JJ by using the more appropriate charge representation [10]. The intermediate *mesoscopic* regime, corresponding to a physical situation in which $E_\mathrm{J} \sim E_\mathrm{C}$, characterizes small-capacitances ($C \sim 1fF$) JJ operating at temperatures $T \sim 10 \div 100$ mK, where the thermal energy $K_\mathrm{B}T$ is lower than the junction energies E_J and E_C [10,11]. In order to exploit the fully quantum-mechanical nature of mesoscopic Josephson junctions, quite recently many authors have studied the interaction of such a mesodevices with non classical electromagnetic fields [12–19]. In this context, the occurrence of current spikes, called quantum Shapiro steps (QSS), in the *I*-*V* characteristic of the mesojunction irradiated by a single-mode low-intensity coherent electromagnetic field has been predicted [12,18,19]. Moreover, it has been proved that the dynamics of a mesojunction exposed to a monochromatic quantized electromagnetic field exhibits an high sensitivity to the quantum coherences of the radiation field state [20,21].

2 The physical system

The system here analyzed consists of a dc-voltage biased mesoscopic junction coupled to an external quantized single-mode far-infrared ($\omega_1 \sim 2\pi \times 10^{14}$ Hz) coherent field. Let's suppose that the initial density matrix $\rho(0)$ describing the combined field-junction system may be factorized as

$$\rho(0) = \rho_\mathrm{J}(0) \otimes \rho_\mathrm{F}(0), \tag{1}$$

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where $\rho_J(0)$ [$\rho_F(0)$] describes the material subsystem [radiation field] at $t = 0$, and $\rho_F(0) = |\alpha\rangle\langle\alpha|$ with $|\alpha\rangle = ||\alpha|\exp(i\theta_\alpha)\rangle$ and $|\alpha| \sim 1$.

It is very difficult to claim for the preparation of a Josephson junction in a specified state. Nevertheless, in the context of many experimental schemes involving these mesodevices, one is mainly interested in the expectation values of suitable physical quantities rather than in the knowledge of $\rho_J(0)$ in its own.

In this paper we show how to acquire information on $\mathrm{Tr}[\rho_J(0)\sin\varphi]$ and $\mathrm{Tr}[\rho_J(0)\cos\varphi]$ at $t = 0$ *without knowing in advance the form of* $\rho_J(0)$. Besides its intrinsic interest, our proposal provides the first essential step of a procedure aimed at detecting the intensity $|\alpha|$ and the phase θ_α of an unknown e.m. coherent state by means of a mesoscopic JJ. The method we are going to present relies on the possibility of measuring the first two quantum Shapiro steps in the I-V characteristic of a mesoscopic JJ exposed to quantized single-mode electromagnetic fields.

3 The Hamiltonian model and the supercurrent time evolution

It is well known that, adopting the Voltage Bias Model and neglecting the quasiparticle tunneling, the Hamiltonian of the junction-field system can be cast in the following form [21,22]:

$$H=\frac{[q+C(V_0+V_F)]^2}{2C}+E_J(1-\cos\varphi)+\hbar\omega_1\left(a^\dagger a+\frac{1}{2}\right). \quad (2)$$

Here $V_F = |E|d = i\sqrt{(\frac{\hbar\omega_1}{2C_F})}(a - a^\dagger)$ is the non classical electromotive force imposed on the superconductive device and V_0 is the dc voltage bias.

Let's begin by considering the Heisenberg equations of motion for the representative operators q, φ, a and $a^\dagger$ of the combined system:

$$I \equiv -\frac{\partial q}{\partial t} = I_{cr}\sin\varphi \quad (3)$$

$$\frac{\partial\varphi}{\partial t} = \frac{2e}{\hbar}\left(\frac{q}{C} + V_0 + V_F\right) \quad (4)$$

$$\frac{\partial a}{\partial t} = -i\omega_1 a - (q + CV_0)\sqrt{\frac{\omega_1}{2\hbar C_F}} - i\frac{\omega_1}{2}\frac{C}{C_F}(a - a^\dagger) \quad (5)$$

$$\frac{\partial a^\dagger}{\partial t} = i\omega_1 a^\dagger - (q + CV_0)\sqrt{\frac{\omega_1}{2\hbar C_F}} - i\frac{\omega_1}{2}\frac{C}{C_F}(a - a^\dagger). \quad (6)$$

This complicated operator coupled differential equations system cannot be analytically solved. However, as showed in references [12,21], in the external field approximation the equation for the operator φ may be approximately reduced to the form:

$$\dot\varphi \approx \frac{2e}{\hbar}\left[V_0 + i\sqrt{\frac{\hbar\omega_1}{2C_F}}(a e^{-i\omega_1 t} - a^\dagger e^{i\omega_1 t})\right]. \quad (7)$$

Integrating equation (7) is now an easy matter and gives

$$\varphi(t) \approx \varphi_0 + \frac{2e}{\hbar}\left[V_0 t - \sqrt{\frac{\hbar}{2\omega C_F}}(a e^{-i\omega_1 t} + a^\dagger e^{i\omega_1 t})\right], \quad (8)$$

where $\varphi_0 = \varphi(t = 0)$ is the phase operator in the Schrödinger picture. Substituting $\varphi(t)$ into equation (3) immediately enables to find the explicit form of the time evolution of supercurrent operator $I(t)$ as

$$\frac{I(t)}{I_{cr}} \approx \sin[\varphi_0 + \omega_0 t - \xi(a e^{-i\omega_1 t} + a^\dagger e^{i\omega_1 t})]. \quad (9)$$

Here $\hbar\omega_0 = 2eV_0$ and $\xi = \frac{\sqrt{2}e}{\sqrt{\hbar\omega_1 C_F}}$ is a real number related to the capacitive parameter depending on the barrier thickness ($d \sim 1$ nm) and the field frequency ω_1.

In order to evaluate $\langle I(t)\rangle = \mathrm{Tr}[\rho(0)I(t)]$, it is useful to cast the supercurrent operator in the form

$$I(t) = I_{cr}\mathrm{Im}\{e^{i\varphi(t)}\}. \quad (10)$$

Taking into account that $\mathrm{Tr}[\mathrm{Im}[e^{i\varphi(t)}]] = \mathrm{Im}[\mathrm{Tr}[e^{i\varphi(t)}]]$, the supercurrent operator expectation value $\langle I(t)\rangle$ assumes the following form

$$\langle I(t)\rangle = I_{cr}e^{-\frac{\xi^2}{2}} \times\Big(\sin[\omega_0 t + 2\xi|\alpha|\sin(\omega_1 t - \theta_\alpha - \frac{\pi}{2})]\mathrm{Tr}[\rho_J(0)\cos\varphi_0] + \cos[\omega_0 t + 2\xi|\alpha|\sin(\omega_1 t - \theta_\alpha - \frac{\pi}{2})]\mathrm{Tr}[\rho_J(0)\sin\varphi_0]\Big). \quad (11)$$

Equation (11), valid in the time interval $[\omega_1^{-1}, \Omega_p^{-1}]$ [12,21], reveals sensitivity of the supercurrent crossing a mesojunction to the intensity $|\alpha|$ and the phase θ_α of the coherent single-mode field which it is coupled to. Unfortunately, such a circumstance, due to the impossibility of measuring currents oscillating at very-high frequencies, does not enable detection of the field parameters $|\alpha|$ and θ_α. Nevertheless, exploiting the Fourier-Bessel expansion of $\sin[\omega_0 t + 2\xi|\alpha|\sin(\omega_1 t - \theta_\alpha - \frac{\pi}{2})]$ and $\cos[\omega_0 t + 2\xi|\alpha|\sin(\omega_1 t - \theta_\alpha - \frac{\pi}{2})]$, equation (11) may be cast in the following form:

$$\frac{\langle I(t)\rangle}{I_{cr}} = e^{-\frac{\xi^2}{2}}\Bigg(\mathrm{Tr}[\rho_J(0)\cos\varphi_0]\sum_{k=-\infty}^{\infty}J_k(2\xi|\alpha|)S(t) + \mathrm{Tr}[\rho_J(0)\sin\varphi_0]\sum_{k=-\infty}^{\infty}J_k(2\xi|\alpha|)C(t)\Bigg) \quad (12)$$

where $S(t) = \sin[(\omega_0 + k\omega_1)t - k\theta_\alpha - k\frac{\pi}{2}]$ and $C(t) = \cos[(\omega_0 + k\omega_1)t - k\theta_\alpha - k\frac{\pi}{2}]$.

It is easy to show that, under the nth resonance condition, defined as $\omega_0 = +n\omega_1$, the expectation value of the supercurrent operator displays a dc component $I_{\rm dc}^{(n)}$, known as nth quantum Shapiro step [12,21], given by the expression:

$$I_{\rm dc}^{(n)} = I_{\rm cr} {\rm e}^{-\frac{\xi^2}{2}} J_n(2\xi|\alpha|)(-1)^n \times\Big(\sin[n(\theta_\alpha + \frac{\pi}{2})]{\rm Tr}[\rho_{\rm J}(0)\cos\varphi_0] + \cos[n(\theta_\alpha + \frac{\pi}{2})]{\rm Tr}[\rho_{\rm J}(0)\sin\varphi_0]\Big) \qquad (13)$$

where $J_n(z)$ is the Bessel function of order n.

It is interesting to compare this result with the well known expression of the nth Shapiro step appearing in the I-V characteristic of a large capacitance ($C \sim 10^{-12}$ F) JJ biased by a combined dc and ac voltage given by $V = V_0 + V_1 \sin(\omega_1 t - \theta_\alpha)$ [22]. To this end, we evaluate the expectation value $\langle V_{\rm F}\rangle$ of the non classical electromotive force with respect the initial system state $\rho(0)$, getting

$$\langle V_{\rm F}\rangle = 2(\frac{\hbar\omega_1}{2C_{\rm F}})^{\frac{1}{2}}|\alpha|\sin(\omega_1 t - \theta_\alpha) \equiv V_1 \sin(\omega_1 t - \theta_\alpha). \qquad (14)$$

This enables the identification $V_1 \equiv 2(\frac{\hbar\omega_1}{2C_{\rm F}})^{\frac{1}{2}}|\alpha|$ from which, in view of the definition of ξ given after equation (9), we arrive at $2\xi|\alpha| = \frac{2eV_1}{\hbar\omega_1}$ that is the expected argument of the Bessel functions in the classical case. Thus, apart from the exponential factor ${\rm e}^{-\frac{\xi^2}{2}}$, whose presence may be traced back to the non commutativity between the creation and annihilation field operators, we claim that equation (13) reduces to the expected classical expression of the nth Shapiro step.

4 Initial expectation values of cosφ and sinφ

Since it is very difficult to claim for the preparation of the JJ in a prefixed state, equation (13) cannot be used to predict $I_{\rm dc}^{(n)}$ since its expression contains the unknown initial mean values of $\sin\varphi$ and $\cos\varphi$. Also, since $I_{\rm cr}$ is of the order of or less than 1 μA only the first QSS in the I-V characteristic of a mesojunction exposed to a very low-intensity coherent field can be practically measured in an accurate way. The structure of equation (13) thus suggests that, if we known the e.m. parameters $|\alpha|$ and θ_α of the field irradiating the JJ, the two n-independent expectation values ${\rm Tr}[\rho_{\rm J}(0)\sin\varphi_0]$ and ${\rm Tr}[\rho_{\rm J}(0)\cos\varphi_0]$ can be read out from quantum current measurements.

It is worth noting that the injected electromagnetic field power is compatible with the predicted appearance of QSS in the Josephson junction characteristic. In fact, the power $P_{\rm F}$ provided by the irradiating field, estimated as $P_{\rm F} = SA \sim 10^{-8}$ Watt, where $S = \frac{1}{2}\epsilon_0 c E^2_{\rm max}$ is the single-photon Poynting vector and $A \sim 0.1$ μm^2 the junction area, is much greater than that required for the QSS appearance. This last power, estimable as $[I_{\rm dc}^{(n)}]^2 R_n$, with $R_n \sim R_Q = 6,5$ kΩ results, in fact, less than $P_{\rm F}$, when steps $I_{\rm dc}^{(n)} < \mu$A are considered.

Our idea is to couple the dc-voltage biased mesojunction, cooled down to a specified temperature between $10 \div 100$ mK, with an e.m. field prepared in a *real coherent state whose intensity* $|\alpha|$ *is a priori known.* If, in such conditions, the amplitude $I_{\rm dc}^{(0)}$ and $I_{\rm dc}^{(1)}$ of the zeroth order and first order QSS respectively are measured, then, in accordance with the prediction expressed by equation (13), we get a linear decoupled system from which ${\rm Tr}[\rho_{\rm J}(0)\sin\varphi_0]$ and ${\rm Tr}[\rho_{\rm J}(0)\cos\varphi_0]$ may be immediately deduced as

$${\rm Tr}[\rho_{\rm J}(0)\sin\varphi_0] = \frac{I_{\rm dc}^{(0)}{\rm e}^{\frac{\xi^2}{2}}}{I_{\rm cr}J_0(2\xi|\alpha|)} \qquad (15)$$

$${\rm Tr}[\rho_{\rm J}(0)\cos\varphi_0] = -\frac{I_{\rm dc}^{(1)}{\rm e}^{\frac{\xi^2}{2}}}{I_{\rm cr}J_1(2\xi|\alpha|)}. \qquad (16)$$

The knowledge of the mean values expressed by equations (15) and (16) does not allow determination of $\rho_{\rm J}(0)$. However, it is worth noting that we know just those properties of the JJ initial state which, in view of equation (13), play an essential role to investigate the structure of the QSS of a JJ irradiated by a weak *coherent field having an arbitrary complex amplitude.*

This means that, measuring in such conditions again the first two QSS amplitudes and inserting their values in the analytical expressions

$$I_{\rm dc}^{(0)} = I_{\rm cr}{\rm e}^{-\frac{\xi^2}{2}} J_0(2\xi|\alpha|){\rm Tr}[\rho_{\rm J}(0)\sin\varphi_0] \qquad (17)$$

$$I_{\rm dc}^{(1)} = -I_{\rm cr}{\rm e}^{-\frac{\xi^2}{2}} J_1(2\xi|\alpha|) \times\Big(\cos\theta_\alpha{\rm Tr}[\rho_{\rm J}(0)\cos\varphi_0] - \sin\theta_\alpha{\rm Tr}[\rho_{\rm J}(0)\sin\varphi_0]\Big) \qquad (18)$$

deduced from equation (13), we can immediately derive the parameters $|\alpha|$ and θ_α of an unknown coherent field.

Let us attempt now an estimate of the amplitudes of $I_{dc}^{(0)}$ and $I_{\rm dc}^{(1)}$ under scrutiny. To this end consider that $|{\rm Tr}[\rho_{\rm J}(0)\cos\varphi_0]| \leq 1$ and $|{\rm Tr}[\rho_{\rm J}(0)\sin\varphi_0]| \leq 1$, so that equations (17) and (18) predict that $I_{\rm dc}^{(0)}$ and $I_{\rm dc}^{(1)}$ are smaller than $I_M^{(0)}$ and $I_{\rm M}^{(1)}$ defined as follows:

$$\frac{I_{\rm M}^{(0)}}{I_{\rm cr}} = {\rm e}^{-\frac{\xi^2}{2}} J_0(2\xi|\alpha|) \qquad (19)$$

$$\frac{I_{\rm M}^{(1)}}{I_{\rm cr}} = 2{\rm e}^{-\frac{\xi^2}{2}} J_1(2\xi|\alpha|). \qquad (20)$$

Figures 1 and 2 report the behaviour of $|I_{\rm M}^{(0)}|$ and $|I_{\rm M}^{(1)}|$ (in unit of $I_{\rm cr}$) respectively as a function of $|\alpha|$, showing the existence of possible values of $I_{\rm dc}^{(0)}$ and $I_{\rm dc}^{(1)}$ compatible with the current experimental setup. It is moreover

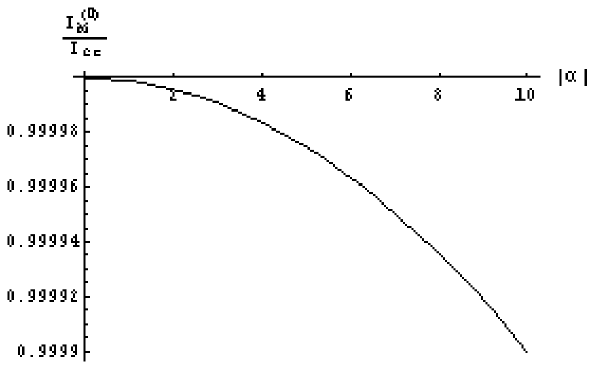

Fig. 1. Plot of $|I_{\rm M}^{(0)}/I_{\rm cr}|$ as a function of $|\alpha|$.

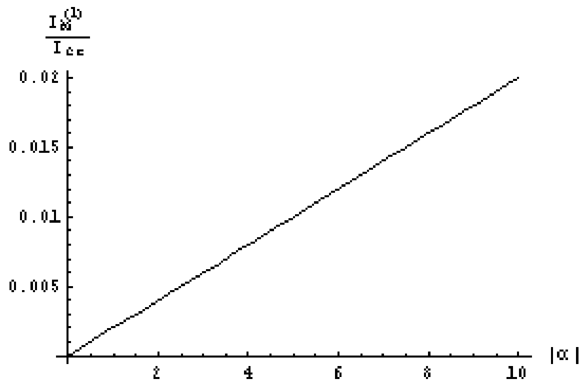

Fig. 2. Plot of $|I_{\rm M}^{(1)}/I_{\rm cr}|$ as a function of $|\alpha|$.

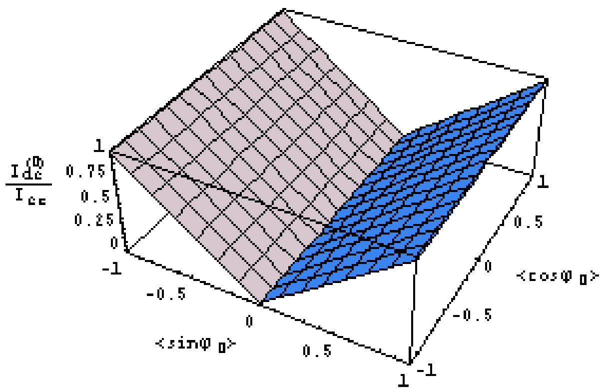

Fig. 3. Plot of $|I_{\rm dc}^{(0)}/I_{\rm cr}|$ as functions of the initial mean values of $\cos\varphi_0$ and $\sin\varphi_0$, in the interval $[-1,1]$, when $|\alpha| = 1$.

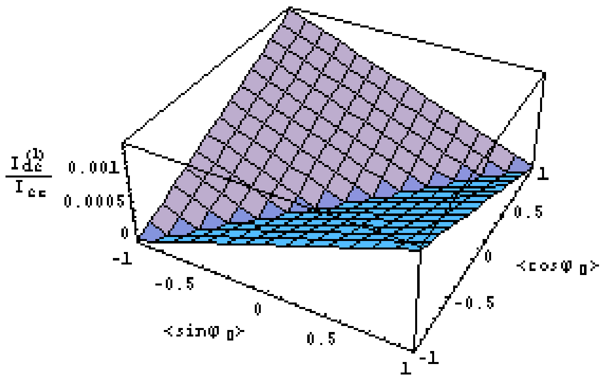

Fig. 4. Plot of $|I_{\rm dc}^{(1)}/I_{\rm cr}|$ as functions of the initial mean values of $\cos\varphi_0$ and $\sin\varphi_0$, in the interval $[-1,1]$, when $|\alpha| = 1$.

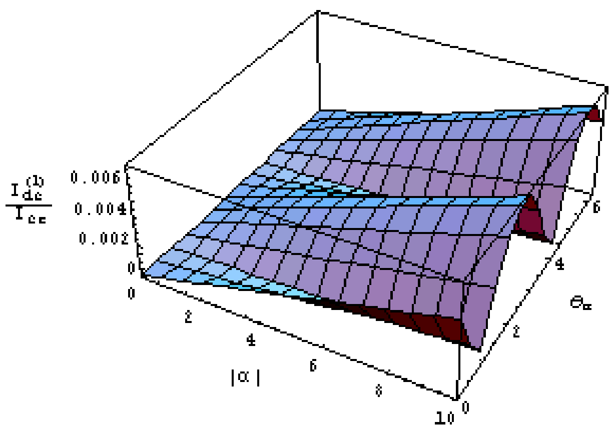

Fig. 5. Plot of $|I_{\rm dc}^{(1)}/I_{\rm cr}|$ as a function of $|\alpha|$ and θ_α when $\langle\sin\varphi_0\rangle = \langle\cos\varphi_0\rangle = 0.5$.

instructive to plot $I_{\rm dc}^{(0)}$ and $I_{\rm dc}^{(1)}$, as given by equations (17) and (18), as functions of the initial mean values of $\cos\varphi_0$ and $\sin\varphi_0$, in the interval $[-1,1]$. Figures 3 and 4 refer to $|I_{\rm dc}^{(0)}|$ and $|I_{\rm dc}^{(1)}|$ (in unit of $I_{\rm cr}$) respectively, when $|\alpha| = 1$. Even these plots confirm the existence of physical conditions wherein the amplitudes of the two first steps turns out to be measurable. Figure 5 finally represents $|I_{\rm dc}^{(1)}/I_{\rm cr}|$ as a function of $|\alpha|$ and θ_α when $\langle\sin\varphi_0\rangle = \langle\cos\varphi_0\rangle = 0.5$.

The dynamical variables describing the time dependent behaviour of the JJ are operators characterized by not negligible quantum fluctuations.

In particular, the fluctuations of the supercurrent operator $\langle(\Delta I)^2\rangle \equiv \langle I^2\rangle - \langle I\rangle^2$, can be explicitly expressed as

$$\langle(\Delta I)^2\rangle = I_{\rm cr}^2[\langle\sin^2\varphi\rangle - \langle\sin\varphi\rangle^2] \tag{21}$$

where $\langle O\rangle \equiv {\rm Tr}[O\rho(0)]$. In analogy with supercurrent operator $I(t)$, we may calculate the expectation value of $I^2(t)$, finding that its nth dc component $\langle I^2\rangle_{\rm dc}^{(n)}$ can be cast in the form

$$\langle I^2\rangle_{\rm dc}^{(n)} = I_{\rm cr}^2\Bigg[\frac{1}{2} - \frac{{\rm e}^{-2\xi^2}}{2}J_{2n}(4\xi|\alpha|)\Big(\langle\cos 2\varphi_0\rangle_{\rm J} \times\cos[2n(\theta_\alpha + \frac{\pi}{2})] - \langle\sin 2\varphi_0\rangle_{\rm J}\sin[2n(\theta_\alpha + \frac{\pi}{2})]\Big)\Bigg]. \tag{22}$$

Analyzing equation (22) it turns out that, coupling the dc-voltage biased mesojunction whit an e.m. field prepared in a real coherent state $|\alpha\rangle = ||\alpha|\rangle$, the following relation

$$J_0(4\xi|\alpha|)[2\langle I^2\rangle_{\rm dc}^{(1)} - I_{\rm cr}^2] = J_2(4\xi|\alpha|)[I_{\rm cr}^2 - 2\langle I^2\rangle_{\rm dc}^{(0)}] \quad (23)$$

between quantities related to zeroth order QSS and first order QSS does exist. It is of relevance that equation (23) establishes a connection between quantities like $\langle I^2\rangle_{\rm dc}^{(0)}$ and $\langle I^2\rangle_{\rm dc}^{(1)}$ which, in principle, may be deduced from measurements of the quantum fluctuations associated to the zeroth order and first order Shapiro steps amplitudes. In this way, such a relation may be tested in laboratory validating the approximated treatment reported in this paper or, in the negative case, inducing to reconsider some delicate aspects of this approach, as for example the external field approximation on which the derivation of the explicit expression of the Heisenberg operator $I(t)$ is based.

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